Chapter 4
Bandpass signaling principles and circuits

Introduction

In this section, we will answer the questions:

- **What is a general representation for bandpass digital and analog signals?**
- **How do we represente a modulated signal?**
- **How do we represent bandpass noise?**

4.1 Complex envelope representation

Modulation

- The process of imparting the source information onto a bandpass signal with a carrier frequency f_c by the introduction of amplitude or phase perturbations or both.

The modulation may be visualized as a mapping operation that maps the source information onto the bandpass signal.

The modulated signals representation

$$v(t) = R(t)\cos\left[\omega_c t + \theta(t)\right]$$

$$v(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

$$x(t) = R(t)\cos\theta(t)$$

$$y(t) = R(t)\sin\theta(t)$$

$$v(t) = \mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\}$$

$$\begin{aligned} g(t) &= R(t)e^{j\theta(t)} \\ &= R(t)\left[\cos\theta(t) + j\sin\theta(t)\right] \\ &= x(t) + jy(t) \end{aligned}$$

Theorem

Any physical bandpass waveform can be represented by

$$v(t) = \mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\}$$

Two other equivalent representations:

$$v(t) = R(t)\cos\left[\omega_c t + \theta(t)\right]$$

$$v(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

4.2 Representation of modulated signals

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❖ The modulated signal is represented by

$$s(t) = \mathrm{Re}\left\{ g(t) e^{j\omega_c t} \right\}$$

where the complex envelope $g(t)$ is a function of the modulating signal $m(t)$:

$$g(t) = g\left[m(t) \right]$$

$g[.]$ performs a mapping operation on $m(t)$

Properties of complex envelope g(t):

$$g(t) = x(t) + jy(t) = R(t)e^{j\theta(t)}$$

$$x(t) = \operatorname{Re}\{g(t)\} = R(t)\cos\theta(t)$$

$$y(t) = \operatorname{Im}\{g(t)\} = R(t)\sin\theta(t)$$

$$R(t) = |g(t)| = \sqrt{x^2(t) + y^2(t)}$$

$$\theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$

4.3 Spectrum of bandpass signals

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a bandpass waveform $v(t)=\mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\}$

if $v(t)\leftrightarrow V(f)$, $g(t)\leftrightarrow G(f)$ ⇒ then $V(f)$ ~?~ $G(f)$, $p_v(f)$ ~?~ $p_g(f)$

the spectrum of the bandpass waveform is

$$V(f)=\frac{1}{2}\left[G(f-f_c)+G^*(-f-f_c)\right]$$

the PSD of the waveform is

$$p_v(f)=\frac{1}{4}\left[p_g(f-f_c)+p_g(-f-f_c)\right]$$

4.3 Spectrum of bandpass signals

the PSD of the waveform is

$$p_v(f) = \frac{1}{4}\left[p_g(f - f_c) + p_g(-f - f_c)\right]$$

- **The total average normalized power of a bandpass waveform v(t) is**

$$\begin{aligned} P_v &= \left\langle v^2(t) \right\rangle \\ &= \int_{-\infty}^{+\infty} p_v(f)\,df \\ &= R_v(0) \\ &= \frac{1}{2}\left\langle \left| g(t) \right|^2 \right\rangle \end{aligned}$$

Peak envelope power

- The **peak envelope power (PEP)** is the average power that would be obtained if |g(t)| were to be held constant at its peak value

- The normalized PEP is given by

$$P_{PEP} = \frac{1}{2}\left[\max\left|g(t)\right|\right]^2$$

Evaluation of power

- ***Example***

  Evaluate the magnitude spectrum for an amplitude-modulated (AM) signal.

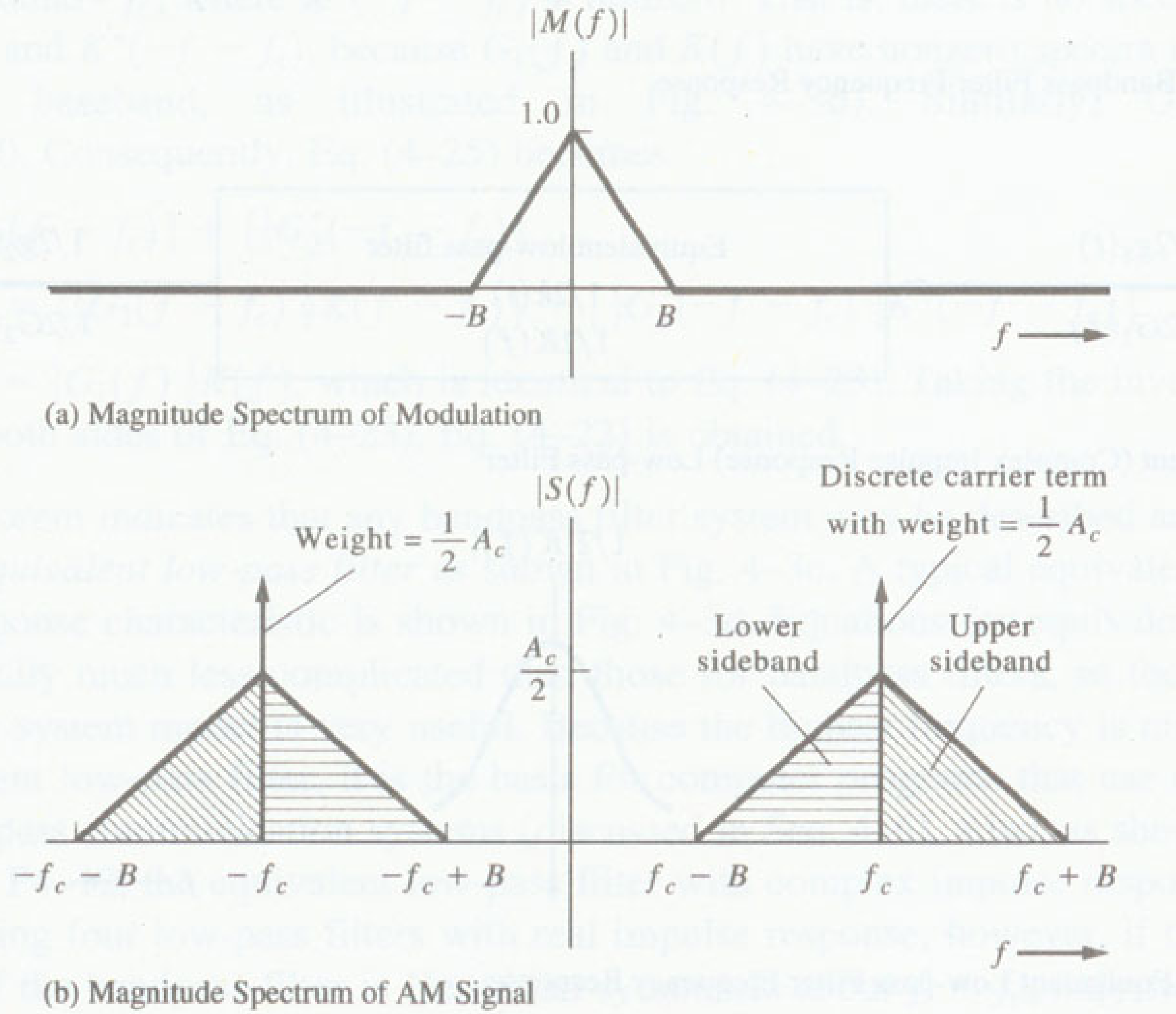
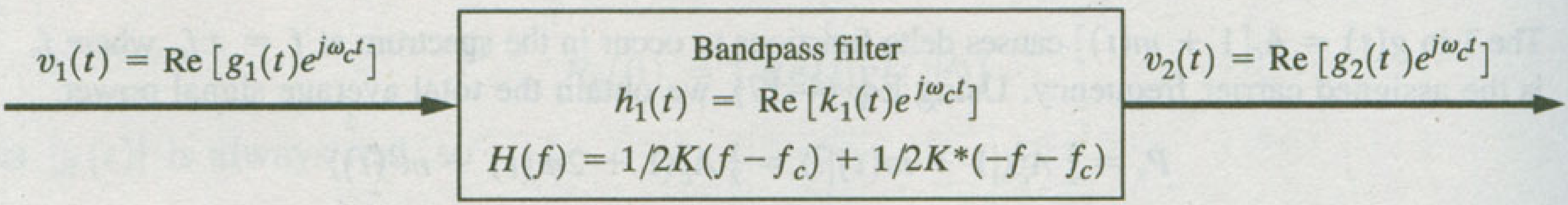


Figure 4–2 Spectrum of AM signal.

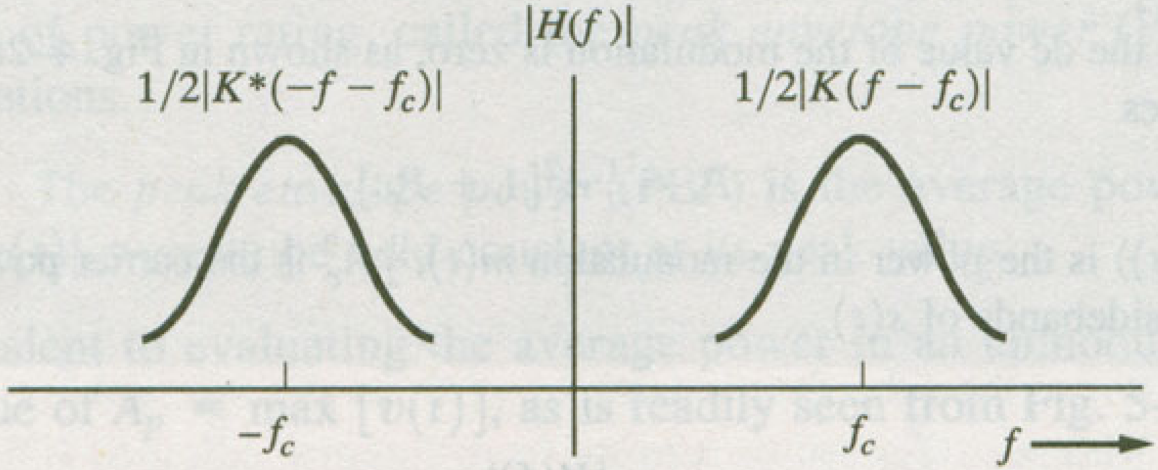
4.5 Bandpass filtering and linear distortion

Bandpass filtering and linear distortion

Equivalent Low-pass filter



(a) Bandpass Filter



(b) Typical Bandpass Filter Frequency Response

Bandpass filtering and linear distortion

Theorem

- The **complex envelopes** for the input, output, and impulse response of a bandpass filter are related by

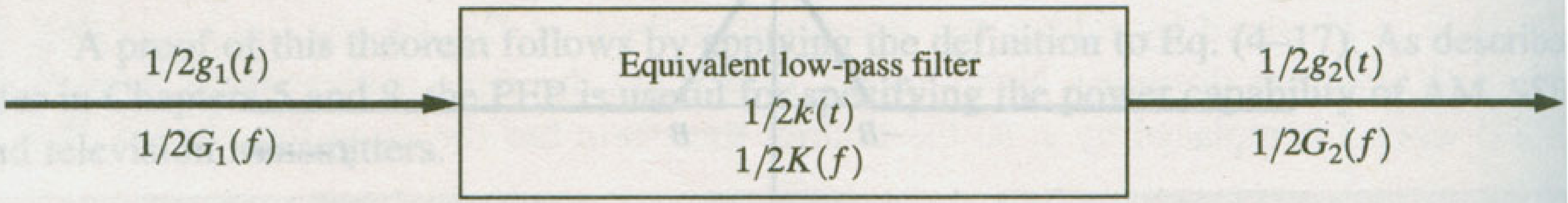
$$\frac{1}{2}g_2(t) = \frac{1}{2}g_1(t) * \frac{1}{2}k(t)$$

where $g_1(t)$ is the complex envelope of the input and $k(t)$ is the complex envelope of the impulse response. It also follows that

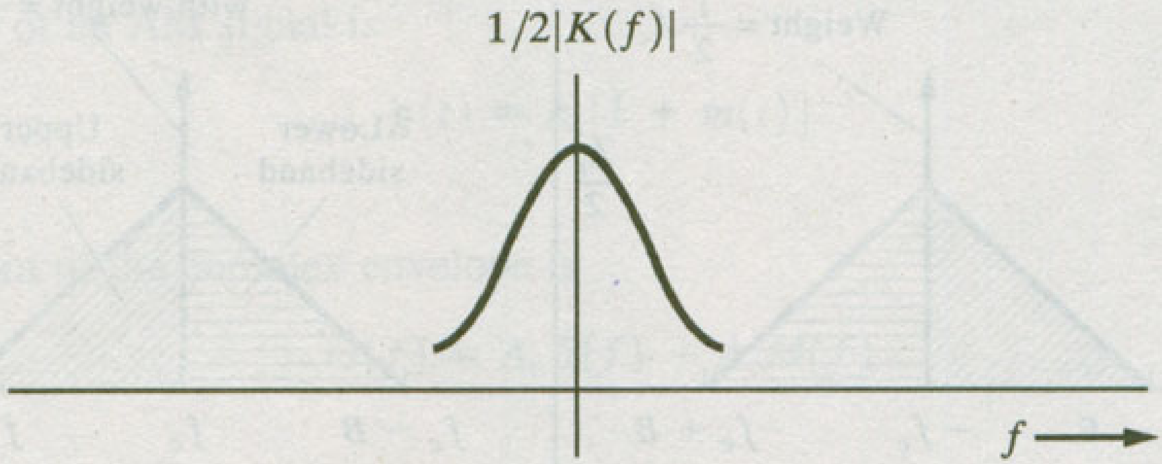
$$\frac{1}{2}G_2(f) = \frac{1}{2}G_1(f)\frac{1}{2}K(f)$$

Bandpass filtering and linear distortion

Equivalent Low-pass filter



(c) Equivalent (Complex Impulse Response) Low-pass Filter



(d) Typical Equivalent Low-pass Filter Frequency Response

Linear distortion

- To have **no distortion** at the output of a **linear time-invariant system**, two requirements must be satisfied:
  - **The amplitude response is flat. That is,**

$$\left| H(f) \right| = A$$

A: constant

  - **The phase response is a linear function of frequency. That is,**

$$\theta(f) = \angle H(f) = -2\pi f T_d$$

Linear distortion

- **For distortionless transmission of bandpass signals, the channel transfer function** $H(f)=|H(f)|e^{j\theta(f)}$ **needs to satisfy the following requirements:**

  - **The amplitude response is constant.**

    $$|H(f)| = A \qquad (4\text{-}27a)$$

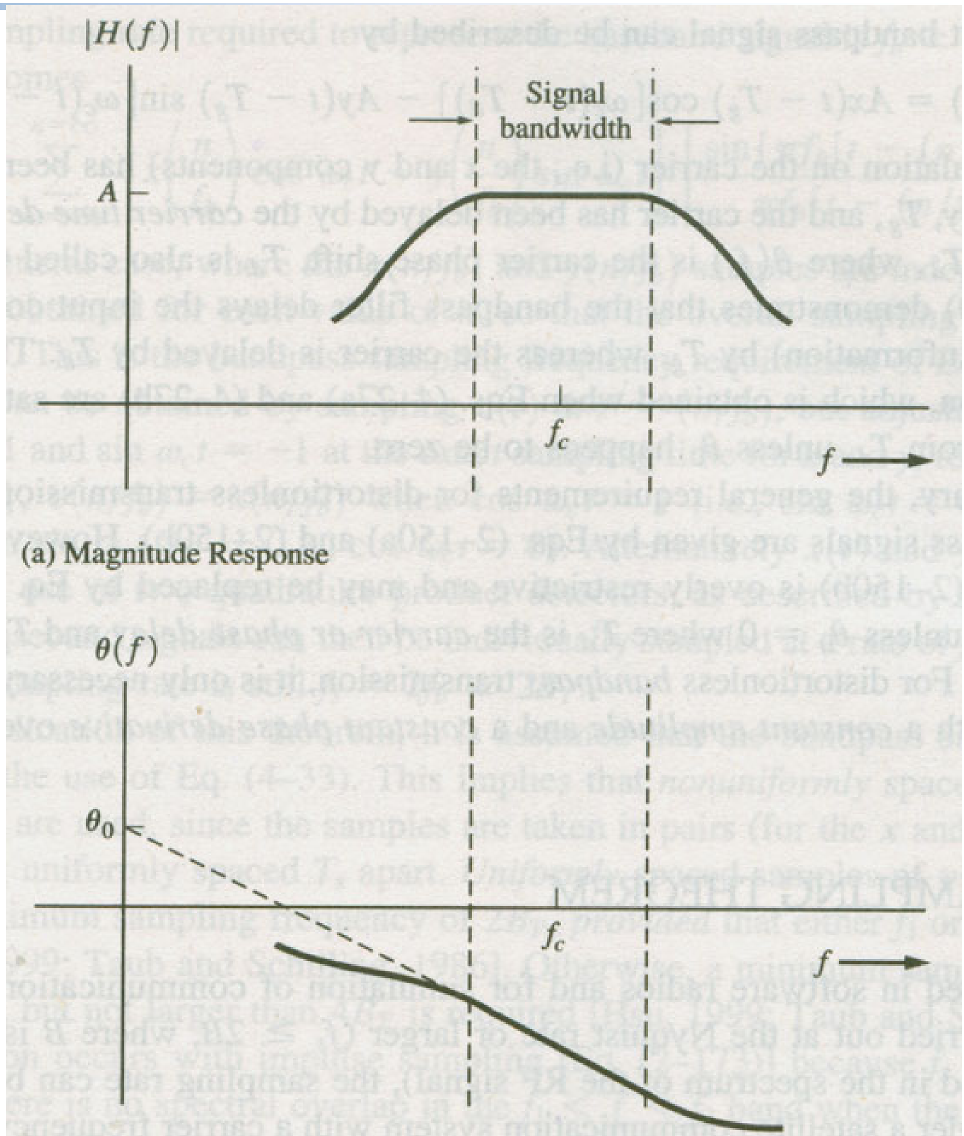
  - **The derivative of the phase response is a constant.**

    $$-\frac{1}{2\pi}\frac{d\theta(f)}{df} = T_g \qquad (4\text{-}27b)$$

Linear distortion

Note:

The Eqs. (4-27a) and (4-27b) are only sufficient requirements for distortionless transmission of bandpass signals.



(a) Magnitude Response

(b) Phase Response

Linear distortion

- The output bandpass signal can be described by

$$v_2(t) = Ax(t-T_g)\cos\left[\omega_c(T-T_d)\right] - Ay(t-T_g)\sin\left[\omega_c(t-T_d)\right]$$

- The bandpass filter delays the input complex envelope (i.e., the input information) by T_g , whereas the carrier is delayed by T_d .

- ***Note:*** T_g will differ from T_d , unless θ_0 happens to be zero.

Linear distortion

summary

- **The general requirements for distortionless transmission of either baseband or bandpass signals are given by Eqs. (2-150a) and (2-150b).**

- **However, for the bandpass case, Eq.(2-150b) is overly restrictive and may be replaced by Eq.(4-27b).**

- **For distortionless bandpass transmission, it is only necessary to have a transfer function with a constant amplitude and a constant phase derivative over the bandwidth of the signal.**

4.6 Bandpass sampling theorem

Bandpass sampling Theorem

- If a (real) bandpass waveform has a nonzero spectrum only over the frequency interval $f_1 < | f | < f_2$, where the transmission bandwidth B_T is taken to be the absolute bandwidth $B_T = f_2 - f_1$, then the waveform may be reproduced form sample values if the sampling rate is

$$f_s \geq 2B_T$$

4.6 Bandpass sampling theorem

- *Proof* .
- **the quadrature bandpass representation is**

$$v(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

- **sampling theorem**

$$v(t) = \sum_{n=-\infty}^{\infty}\left[x\left(\frac{n}{f_b}\right)\cos\omega_c t - y\left(\frac{n}{f_b}\right)\sin\omega_c t\right]\left[\frac{\sin\{\pi f_b(t-(n/f_b))\}}{\pi f_b(t-(n/f_b))}\right]$$

- **For the general case, where the $x\ (n/f_b)$ and $y\ (n/f_b)$ samples are independent, two real samples are obtained for each value of n, so that the overall sampling rate for $v(t)$ is** $f_s = 2f_b \geq 2B_T$

Example SA 4.5

The signal s(t) is to be sampled by using any one of three methods, for each of three sampling methods, determine the minimum sampling frequency required.

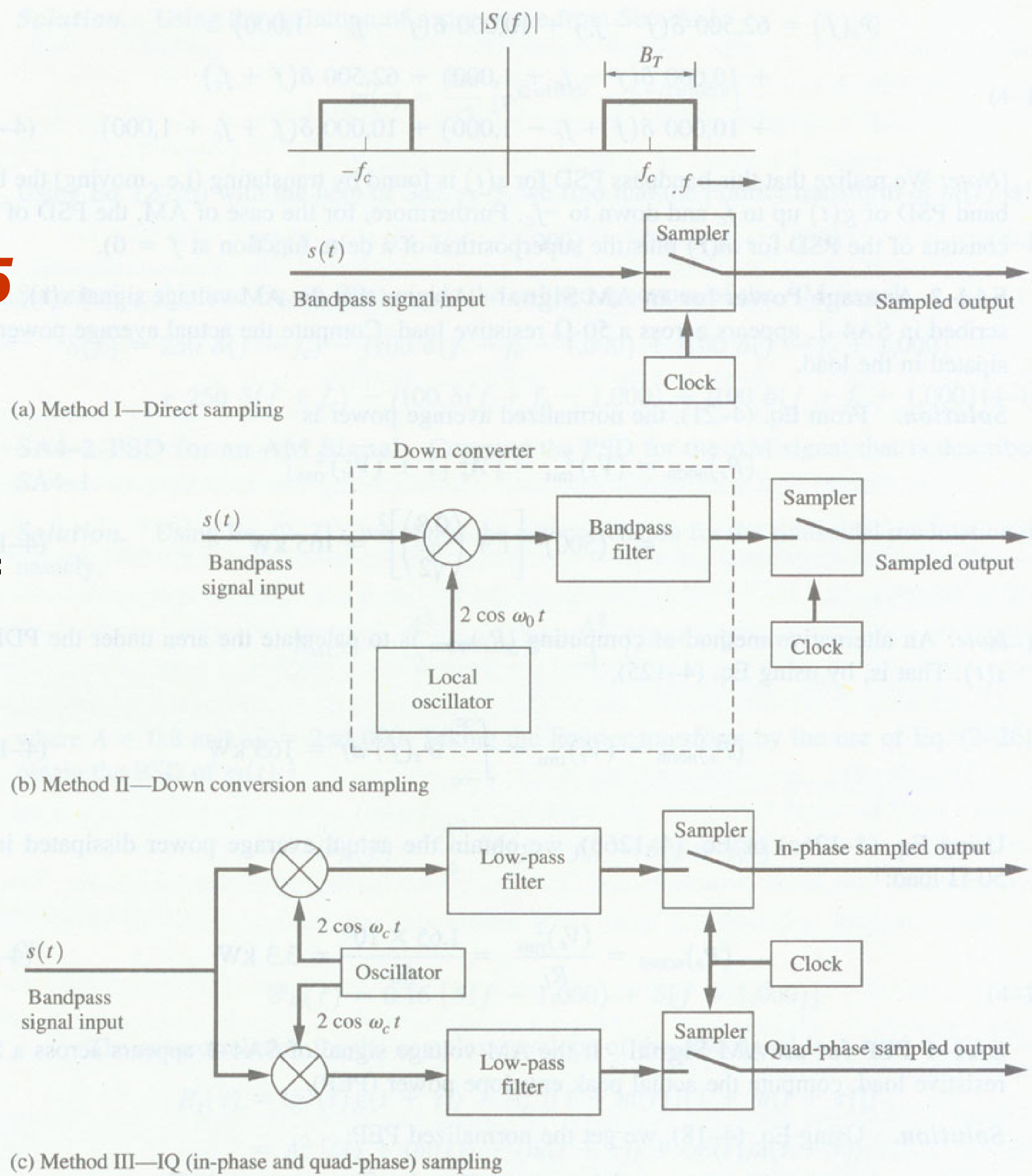


Figure 4–32 Three methods for sampling bandpass signals.

❖ *Bandpass dimensionality theorem*

Assume that a bandpass waveform has a nonzero spectrum only over the frequency interval $f_1 < | f | < f_2$, where the transmission bandwidth B_T is taken to be the absolute bandwidth given by $B_T = f_2 - f_1$ and $B_T << f_1$, the waveform may be completely specified over a T_0-second interval by

$$N = 2B_T T_0$$

independent pieces of information. N is said to be the number of dimensions required to specify the waveform

4.7 Received signal plus noise

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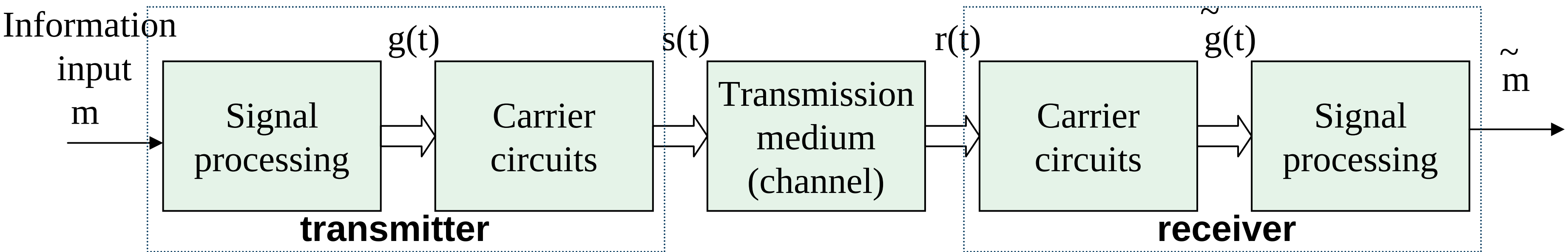

The signal out of the transmitter is

$$s(t) = \mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\}$$

The received signal plus noise is

$$r(t) = s(t) * h(t) + n(t)$$

If the channel is distortionless, $H(f) = Ae^{j(-2\pi f T_g + \theta_0)} = (Ae^{j\theta_0})e^{-j2\pi f T_g}$

the signal plus noise at the receiver input is

$$r(t) = \mathrm{Re}\left\{Ag(t - Tg)e^{j(\omega_c t + \theta(f))}\right\} + n(t)$$

4.7 Received signal plus noise

- If the receiver circuits are designed to make errors due to the **channel group delay (T_g) and $\theta(f)$ effects negligible**, we can consider the signal plus noise at the receiver input to be

$$r(t) = \mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\} + n(t)$$

where the effects of channel filtering, if any, are included by some modification of the complex envelope $g(t)$.